

The Istat Microsimulation Models

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Sommario

Obiettivo di questo lavoro è di inserire il modello di microsimulazione sulle famiglie sviluppato dall'Istat all'interno dell'evoluzione dei modelli statici e comportamentali. Oltre a una panoramica internazionale sui modelli si descrivono le implicazioni dei modelli statici per l'analisi delle policy.

Parole chiave: microsimulazione, tassazione.

Abstract

The aim of this paper is to relate the new microeconomic model on households developed by Istat to the development of the static and behavioural models presented in the literature. Both a survey on the international experiences and a focus on the implication of the static model for policy evaluation are presented.

Keywords: Microsimulation, taxation.

1. Introduction

The Istat new microsimulation models described in this volume come out at a moment of maturity of microsimulation research, when the respective roles of static and behavioural models - and their relationships and interactions - have been made clear and productive, after decades of encounters, conflicts and re-encounters. It is instructive to summarize the process that brought us where we stand now (Section 2). Then we will look at the current state-of-the-art in static modelling in Italy and elsewhere (Section 3). In Section 4, we address the issue of how to interpret the static microsimulation results from the policy point-of-view. We also suggest some procedures that have the potential of enriching the static models with elements of behavioural response without having to develop a fully specified structural behavioural model. Section 5 contains the conclusions.

2. The peculiar evolution of microsimulation

The first proposal for a «microsimulation model» appears in Orcutt (1957). More than an

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academic idea, it is a proposal addressed to the policy makers. At the time, the models used for policy analysis were macro models consisting of (mostly accounting) relationships among aggregates, with little and vague micro foundations. Orcutt's critique is focussed on four points: policy relevance, aggregation, micro foundations and representation of behaviour.

Policy Relevance

"Existing models of our socio-economic system have proved to be of rather limited predictive usefulness. This is particularly true with respect to predictions about the effects of alternative governmental actions [...]."

Micro-foundations

[...] "research efforts in the behavioral sciences have yielded and show promise of yielding very substantial amounts of knowledge about elemental decision-making units. However, existing models of socio-economic systems are neither built in terms of such units nor are they well adapted to making use of knowledge about such units".

[...] "The most distinctive feature of this new type of model is the key role played by actual decision making units of the real world such as the individual, the household and the firm".

Aggregation

"[...] current models of our socio-economic system only predict aggregates and fail to predict distributions of individuals, households, or firms [...]"

"Aggregation of relationships about elemental decision-making units is fairly easy if the relationships to be aggregated are linear [...]. However, if nonlinear relationships are present, then stable relationships at the micro level are quite consistent with the absence of stable relationships at the aggregate level".

Representation of behaviour

"This new type of model consists of various sorts of interacting units which receive inputs and generate outputs [...] Probability distributions specify the probabilities associated with the possible outputs of the unit.

"Prediction about aggregates would still be needed but will be obtained by aggregating behavior of elemental units..."

Summing up:

Policy-relevant models should be based on a disaggregated and explicit representation of the micro-units and of their interactions. Micro choices are represented as probabilistic events. The probabilistic representation of behaviour naturally suggests simulation as the tool to solve the model: thus, micro-simulation. A very ambitious project indeed: early improvements at the end of the 50s in micro data collection and management and in digital computing made Orcutt confident in the feasibility of the project. Realistically, the project had to be articulated in specific building blocks or modules.

Orcutt and associates proceeded to the implementation phase (Wisconsin, Urban Institute, Yale) working in particular on the household sector and on socio-demographic dynamics. A summary of early implementations is provided by Orcutt et al. (1961).

The story that follows can be divided into four periods: Conflictual marriage, Divorce, Preparing for a re-encounter, Re-marriage.

Conflictual marriage

In principle, Orcutt's proposal would have represented an ideal match between policy-relevant modelling and microeconomics (or micro-analytic behavioural theories in general). However, the two partners were not ready for that. Orcutt – a background in engineering and physics - had probably little confidence in microeconomic theory. Orcutt's project did not receive much interest either by microeconomists or by econometricians. Appropriate, policy relevant, empirical specifications of microeconomic models (i.e. microeconomic models) were not available yet.

As a consequence, the behavioural relationships illustrated for example in Orcutt et al. (1961) are reduced form specifications. This approach apparently contradicts Marshack (1953), whose lesson essentially tells that in order to be able to give policy prescription you need structural models or at least estimates of policy-invariant parameters. It must be noted that Orcutt wrote in a period where the empirical design of microeconomic policies was absent: therefore, the issues related to budget sets modifications implied by micro-policies were not on the agenda. In any case, probably Orcutt and associates thought that the most urgent destination of research efforts and resources was the exploitation of newly available micro data and computing resources.

Divorce

During the 70s, 80s and 90s, large microsimulation models in various countries (US, Canada, Scandinavian countries, Australia) acquire popularity, also at the policy making level.

The microsimulation community in this period focusses on the quality of data and the accounting reliability of the predictions. Behavioural responses are left outside. Non-behavioural models are more palatable to policy makers. Large part of the research effort is devoted to tax-benefit simulation models (e.g. EUROMOD).

Preparing for a re-encounter

During the same period that marks a divorce, many developments – at various levels: policy, theory, empirical methods – take place, preparing for a more mature and fruitful re-encounter.

- At the policy level, starting with the mid 60s, there is an increasing interest (war on poverty, tax reforms, welfare reforms etc.) in issues that involve structural changes in the opportunity sets.
- The lesson by Marshack (1953) and Hurvitz (1962) – revived by Lucas (1976) – i.e. you need structural models to make policy simulation, gets eventually fully learnt.
- Heckman (1974), Hausman (1985) and many others develop appropriate models to account for the complexities in the opportunity set (as those implied by newly conceived tax-benefit reforms).
- Discrete choice and random utility models (McFadden 1984) offer new and more flexible tools to estimate and simulate choices subject to complicated constraints.
- Applied microeconomists start using microsimulation techniques to compute responses to policies (Zabalza 1983: possibly the first one). Traditionally, even when using micro data, economists used to compute behavioural responses for an “average” or “representative” individual and ignored the random components: a procedure that can lead to misleading results when the behavioural relationship is non-linear.

Re-marriage

The third millennium marks the re-encounter of microsimulation and microeconomics (-mics)(-metrics). The two partners are now ready.

- An example: Aaberge et al. (2000), presented at the workshop on Microsimulation in the New Millennium, Cambridge, 1999.
- In 2010, ISER (that hosts EUROMOD in Essex) organizes a large workshop on behavioral responses in microsimulation models.
- Compare the program of the IMA conference in Canberra 2003 (most of the papers are arithmetic) to the IMA conference 2013 again in Canberra (most of the papers are behavioural, especially labour supply) and to the last 2014 European IMA conference in Maastricht.

The current period witnesses a clearer vision of the respective roles of non-behavioural and behavioural models, of their possible integration and of new methods to extend the non-behavioural models. For decades, and for good reasons, policy makers remained suspicious about the reliability and generality of behavioural models. At the same time, they have learned to appreciate the value of non-behavioural microsimulation models as robust and invaluable tools. Now they start realizing that some representation of behavioural responses would be important and in some cases not dispensable. As an example, recent discussions about the redesign of income support mechanisms naturally lead us to ask about incentives (to work more or less or to work at all). Issues of this kind require in one way or another a representation of behavioural responses. We might say that static microsimulation models, besides their intrinsic value, have acted as a “benevolent” Trojan horse in channeling the perspective of behavioural modelling.

However, this does not mean that the only way to take is a full integration of static and behavioural models. Especially from the point of view of an institutional research department, a cautious approach is certainly appropriate. First, under many circumstances, static microsimulation is all that is needed. Second, even without adopting a full-blown structural behavioural model, there are various procedures to “enrich” the static microsimulation results and make it possible to produce approximate inferences on behavioural and welfare effects. In Section 4 we will provide a few examples.

3. Microsimulation models in Italy and in the World

I will start with a note on terminology. In the microsimulation community, there is some unnecessary ambiguity in the way different types of models are denominated. My preference would be to distinguish two dimensions: time (static vs. dynamic) and behaviour (behavioural vs. non-behavioural). This type of classification is consistent with the tradition established in economic theory. The analysis of how a consumer’s budget set changes due to changes in prices and or income is non-behavioural (although it might suggest some likely changes in behaviour as well: see Section 4). The same analysis, however, could be static (i.e. it might refer to a permanent scenario in a given period, whatever the length of period) or dynamic (i.e. it might refer to an intertemporal budget set). A static and non-behavioural analysis would investigate how the opportunities or the constraints change due, for example, to population’s ageing or to some exogenous change in consumers’ characteristics or environment. A behavioural analysis would instead include the change in

behaviour as a response to the changed budget set (whether static or dynamic). Outside economics, the expression static behavioural analysis could probably sound weird since behavioural responses need time to materialize: however, what economists refer to in this case is the analysis of an equilibrium configuration of opportunity sets and choices in a given point or period of time. Comparative statics is therefore the analysis of different equilibria: they might take place in different point in time but the analysis is not dynamic, since it is silent upon what happens meanwhile (see Colombino 2013 for a static behavioural simulation procedure that is consistent with the concept of comparative statics). A behavioural dynamic analysis should tell us what happens at different points in time (not necessarily equilibria) that are in a real, not figurative, sequence. Summing up, the classification – with some example – would be as follows:

Table 1 – Microeconometric model classification

	Non-behavioural	Behavioural
Static	e.g. EUROMOD	e.g. ECONLAV
Dynamic	e.g. DYNASIM	e.g. CAPP_DYN

Within the category of behavioural models, we might want to further distinguish between structural models and reduced-form models. Structural models aim at representing choices as function of structural – i.e. policy invariant – parameters (Marschak 1953; Hurwicz 1962). For example, when representing consumers' choices, a structural model would permit a separate identification of preferences (by assumption policy invariant) and opportunity sets (which can be modified by policies). Reduced-form models (e.g. CAPP_DYN) represent choices as functions of parameters that are mixtures of parameters that in general do not allow to identify how they might be affected by policy changes (Lucas 1976). Reduced-form models can provide a very good approximation under the observed policy regime. They may also provide reasonable approximations under policy changes, but that very much depends on the characteristics of the policy changes, and the models' performance is difficult to judge ex-ante. Behavioural structural models are more often developed by academic researchers in view of the analysis of some specific issue – e.g. the static behavioural model of Aaberge and Colombino (2013) or the dynamic behavioural model of Todd and Wolpin (2006) – rather than as general-purpose platforms to be used within an institution. An exception is represented by ECONLAV, a static behavioural – and structural – model (De Luca et al. 2012).

In the microsimulation literature, alternative – and in my view confusing - terminologies are also used:

- Non-behavioural models are often called static or arithmetic.

- Models that are behavioural and/or dynamic, are often called dynamic;

A recent survey describing the design of static non-behavioural models is provided by Li et al. (2014). Previous surveys include Merz (1991), Sutherland (1995) and Citro & Hanushek (1991). For surveys focussed on Europe and on EUROMOD in particular, useful references are Sutherland (1995) and Sutherland & Figari (2013). Besides the new ISTAT models described in this volume, other static non-behavioural Italian models are surveyed by Curiel (2012).

Although we already have many static microsimulation models operating in Italy and a number of researchers have been using them also as algorithms matched to behavioural microeconomic models, the research effort illustrated in this volume is very welcome as

a major step in establishing a sort of “official” platform adopting state-of-the-art methods and best-choice datasets.

4. Static models and policy analysis

Non-behavioural models – by definition – do not account for behavioural responses. However, there are cases when their outputs allows inferences on welfare effects. Moreover, they can be complemented with statistics that are sufficient to make local and approximate inferences on both behavioural and welfare changes, without adopting a full-blown structural behavioural microsimulation approach.

An analogy with meteorological simulations and prediction might be useful. While we have models that explicitly produce the probability of (say) rain with a full-blown structural approach, a more common (and possibly thought to be more robust) procedure, consists of using a model to simulate the basic physical data and then complement them with expert evaluations, previous estimates and relevant statistics in order to generate a prediction of the event ‘rain’.

The basic case

The standard scenario where non-behavioural simulations may be sufficiently informative is when the policies or the reforms can be represented as marginal changes in prices \mathbf{p} and/or in unearned income y . Let $\mathbf{p}'\mathbf{x} = y$ be the consumer budget constraint. Note that the bundle \mathbf{x} might include hours of work (with a corresponding negative price, e.g. $-w$). Let $V(\mathbf{p}, y)$ be the indirect utility function. Let us consider a marginal

change $(d\mathbf{p}, dy)$. Then we have: $dV = [\partial V / \partial \mathbf{p}]' d\mathbf{p} + \mu dy$, where $\mu \equiv \frac{\partial V}{\partial y}$ is the

marginal utility of income. By applying Roy’s Theorem (i.e. $[\partial V / \partial \mathbf{p}] = -\mathbf{x}\mu$) we get:

$\frac{dV}{\mu} = -\mathbf{x}' d\mathbf{p} + dy$. The right-hand side is the change in the budget, conditional on the

pre-reform consumption bundle \mathbf{x} . The left-hand side is the monetary equivalent of the change in utility. Therefore, the result tells us that the change in the consumer’s budget (i.e. the basic result produced by a non-behavioural simulation) is a money-metric measure of the change in utility.

Turning to the production side, let \mathbf{q} be the prices faced by the (price-taker) firm and let $\pi(\mathbf{q})$ be the profit function. Then $d\pi = [\partial \pi / \partial \mathbf{q}]' d\mathbf{q} = \mathbf{x}' d\mathbf{q}$ (due to Hotelling’s Lemma $[\partial \pi / \partial \mathbf{q}] = \mathbf{x}$). The total change in (money-metric) would then be $dW = \mathbf{x}'(d\mathbf{q} - d\mathbf{p}) + dy$. The example clarifies the logic that can guide extensions of static models in view of policy applications. However, it might be of limited practical value. This is so, because of two reasons:

- i) Taxes or subsidies applied under the current policy regimes might make it impossible

to represent the budget constraint and the profit function in the same way as we did above.

ii) In general, policy reforms might involve non-marginal changes.

Even when facing these complications, there are a variety of methods by which we can enrich the static simulation results in order to make approximate inferences upon behavioural changes and welfare effects. We illustrate some of them below.

Harberger-type approximations

Harberger (1964) showed that, in a perfectly competitive market and under mild assumptions, the welfare effect $\frac{dW(t)}{dt}$ of a small change dt of a tax applied to good x can

be approximated by $t \frac{dx}{dt}$. If the change in t is not marginal (e.g. a change from t_0 to t_1), we can integrate the above expression:

$$W(t_1) - W(t_0) = \int_{t_0}^{t_1} t \frac{dx(t)}{dt} dt$$

which in turn could be approximated as a sum of terms $t \frac{dx}{dt}$ evaluated at different values of t in the range (t_0, t_1) . We only need local measures of behavioural response. A textbook application is the “triangle” formula for the consumers’ net welfare change:

$$W(t_1) - W(t_0) = -\frac{1}{2} b (t_1 - t_0)^2$$

where $-b$ is the slope of a linear demand curve.

Chetty’s “sufficient statistics” approach

Chetty (2009) generalizes Harberger’s approach to more interesting cases (heterogeneous agents, discrete choices etc.). The idea essentially consists of complementing non-behavioural computations with “sufficient statistics” of local behavioural response, thus avoiding the need to develop a full-blown structural behavioural model. Note that these methods produce approximate results both on behavioural responses and on welfare effects. As a simple example, given a non-marginal variation of the wage rate, we could approximate the labour supply response by applying previously estimated labour supply elasticities (at the extensive and/or intensive margin, depending on the starting position of the individual). Saez (2001, 2002) has derived optimal tax-benefit formulas that only require local measures of intensive and extensive labour supply elasticities. (Immervoll et al, 2007) provide an empirical application. Of course there is a price to pay when dispensing with the assumptions required by explicit behavioural simulation: the assumptions leading to the theoretical formulas and those underlying the empirical measures of elasticities are in general different and might not be mutually consistent (this problem carries over to Harberger’s approach).

Local measures of incentives

Instead of computing local approximations of behavioural changes, one might simply compute local measures of incentives that are likely to induce changes (Jara and Tumino, 2013). Examples include the computation of Replacement Rates (Immervoll and O'Donoghue, 2004; O'Donoghue, C. 2011), Marginal Effective Tax Rates (Harding and Polette, 1995; Beer, 2003; Creedy et al., 2003; Scholz, 1996; Dolls et al, 2012) and the rate of return to education (O'Donoghue, 1999).

Using discrete opportunities as a menu of potential choices

A different line of attacking the problem consist of considering discrete opportunity sets. This comes natural for example when tax-benefit reforms and labour supply responses are the focus of interest. As an example, it might be natural to assume that each individual can choose among a (small) set of alternatives, such as non-working, part-time and full-time. For each alternative we can compute the net available income given a certain tax-benefit system. For individuals who are observed as not working, we will need to impute (with some missing-values-filling procedure) the gross wage rate. Then, for a generic individual, the alternatives could be described as follow:

$$(L_1, C_1^R), \dots, (L_M, C_M^R)$$

where L_j = "leisure" available if alternative j is chosen and C_j^R = net available income under tax-benefit regime R if alternative j is chosen.

Let k indicate the currently chosen alternative under the current tax-benefit regime $R = 0$. Standard presentations of results would for example consist in running the non-behavioural microsimulation model give the chosen alternative k under a new tax-benefit regime $R = P$ and comparing, say, C_k^P to C_k^0 . A more informative report would be produced by running the model at all the alternative 1, ..., M. This would lead to comparing

$$(L_1, C_1^P), \dots, (L_M, C_M^P)$$

to

$$(L_1, C_1^0), \dots, (L_M, C_M^0).$$

Even without any explicit behavioural modelling or measures, the comparison might suggest likely directions of behavioural responses.

Evaluating discrete opportunity sets with "calibrated" utility functions

This further enrichment builds on the previous one and assumes that we are prepared to use some standard evaluation (utility) function $U = V(L, C) + \varepsilon$. This function could have been previously estimated. If we are not prepared to rely on demanding econometric estimates, the function V could be "calibrated". As a crude simple example, one might consider $V(L, C) = L^\alpha C^{(1-\alpha)}$ and "calibrate" the value of α for example as follows:

$$\alpha = L_k / (T + y / w)$$

where

L_k = observed choice (hours of leisure)

T = total available time

y = gross unearned income

w = gross wage rate.

Note that one could compute a different α for each individual. If the random variable is assumed to have a standard extreme value distribution, then

$\ln \sum_j e^{V(L_j, C_j^R)}$ is a measure of average utility attained under regime R . Moreover,

$\sum_i C_i^R \frac{e^{V(L_i, C_i^R)}}{\sum_j e^{V(L_j, C_j^R)}}$ is a prediction (including behavioural responses) of the net

available income under regime R .

5. Conclusion

From the viewpoint of database, methodology and scope for detailed policy analysis, the new Istat microsimulation models promise to be the most up-to-date official microsimulation platform. Besides the basic utilization as producers of timely non-behavioural simulation, they would also be very useful as algorithms matched to structural microeconomic models. Moreover, I suggested that occasionally they might also be complemented with “sufficient non-behavioural” that, to a certain extent, permit inferences on behavioural and welfare effects without requiring an explicit structural behavioural model.

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